**CIS 431 Exercises on Graphs**

## Minimum Spanning Tree

1. Find the minimum spanning tree using the Dijkstra-Prim algorithm for the each of the graphs in group A. The starting node is A. Please show all steps.

2. Find the minimum spanning tree using the Kruskal algorithm for the each of the graphs in group A. Please show all steps.

3. Prove that if there is one edge with a weight smaller than all of the other edges, that edge will be part of every minimum spanning tree.

4. Prove that if a connected graph has edge weights that are all distinct, i.e., no two edges share the same weight, there is only one minimum spanning tree.

## Shortest Path

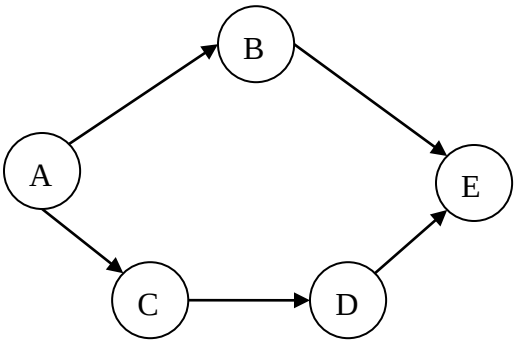
5. Execute the shortest path algorithm for the each of the graphs in group B. The starting node is A. Please create the short-path tree for each of them.

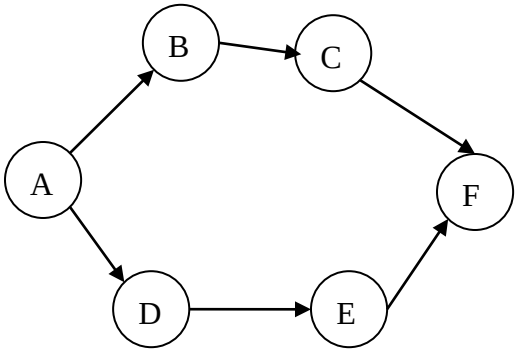
## Bi-connected Component

6. Determine the bi-connected component for the each of the graphs in group C.

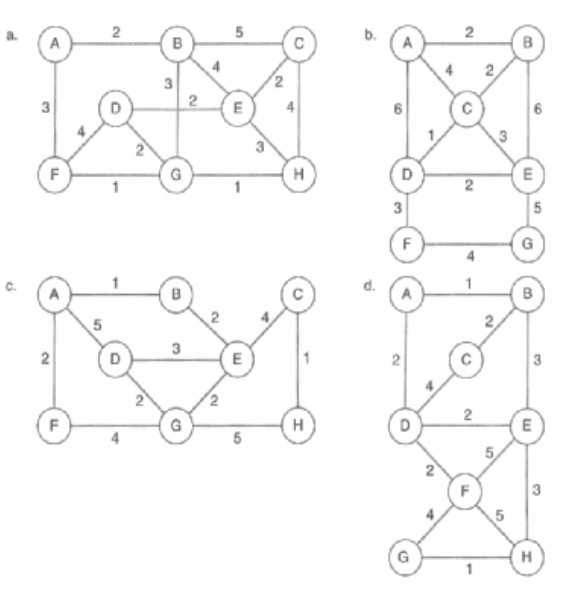
## Topological Ordering

7. Prove if a directed graph G has a topological ordering, then G is a DAG.
8. Prove for every DAG G, there is a node *v* with no incoming edge.
9. Use #8 to prove every DAG G has a topological ordering. Hint: use mathematical induction.
10. Find all topological orderings for the following two DAGs.

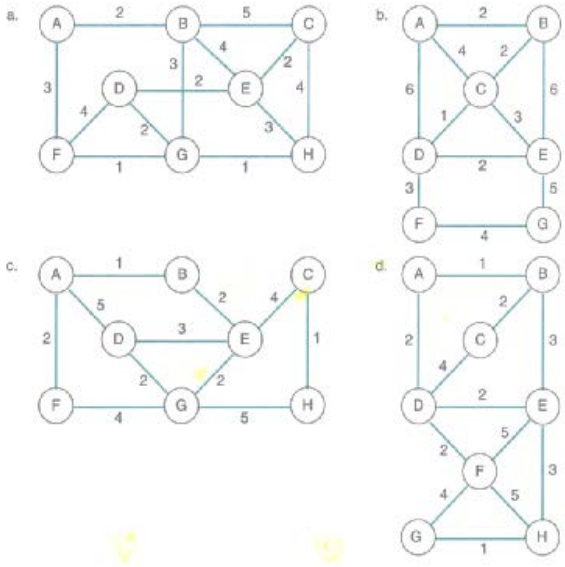


Group A




Group B




Group C